FAR BEYOND

MAT122

Area Between the Curves

Stony Brook University

Integrating with Absolute Value

ex: Evaluate $\int_{-4}^{5} |x|\,dx$ Start by writing as a piecewise function: $f(x) = \begin{cases} -x, & x < 0, \\ x, & x \geq 0. \end{cases}$

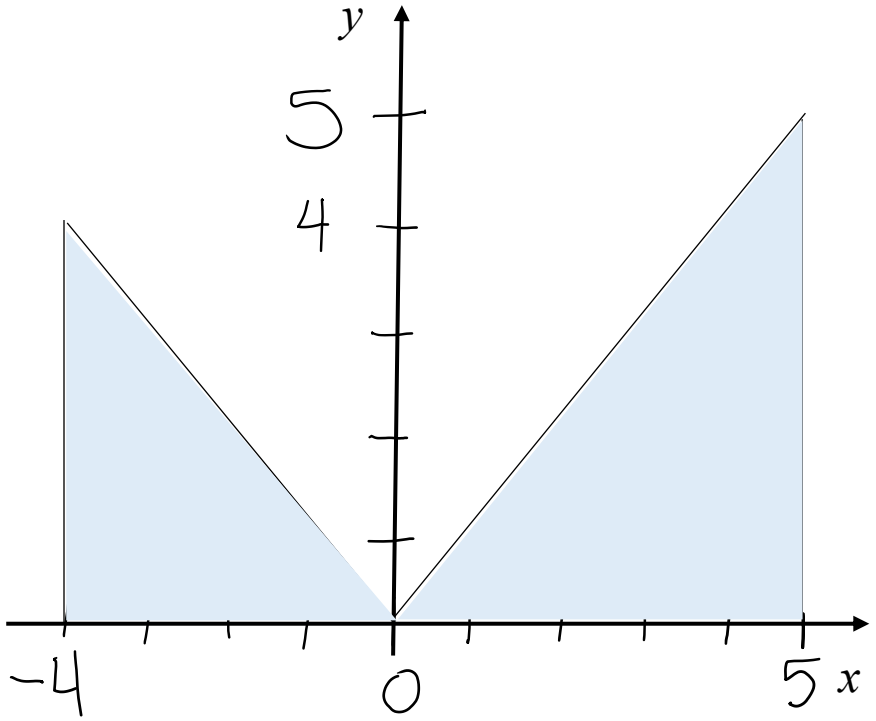
$$= \int_{-4}^{0} -x\,dx + \int_{0}^{5} x\,dx$$

Method #2: use standard shapes to find area

$$= \boxed{\frac{41}{2}}$$

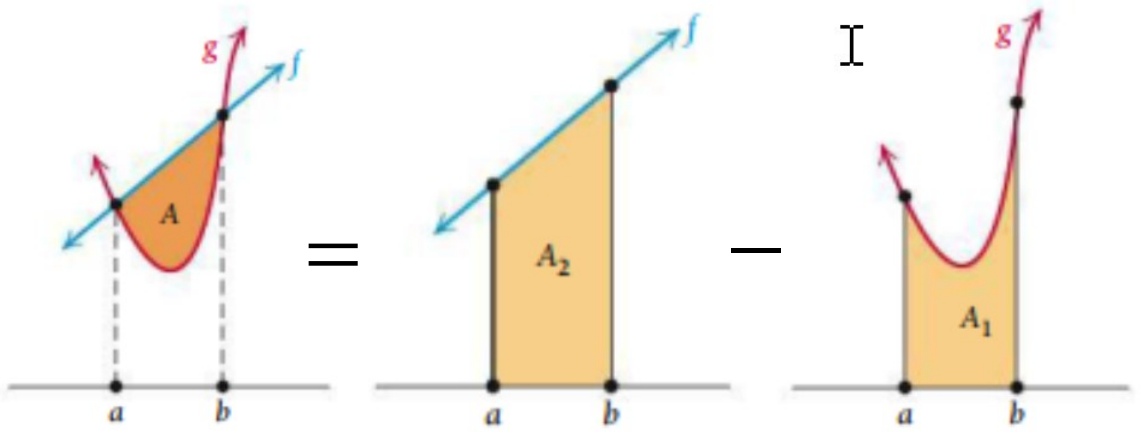
$$A_{\Delta} = \frac{1}{2}bh$$

$$= \boxed{\frac{41}{2}}$$

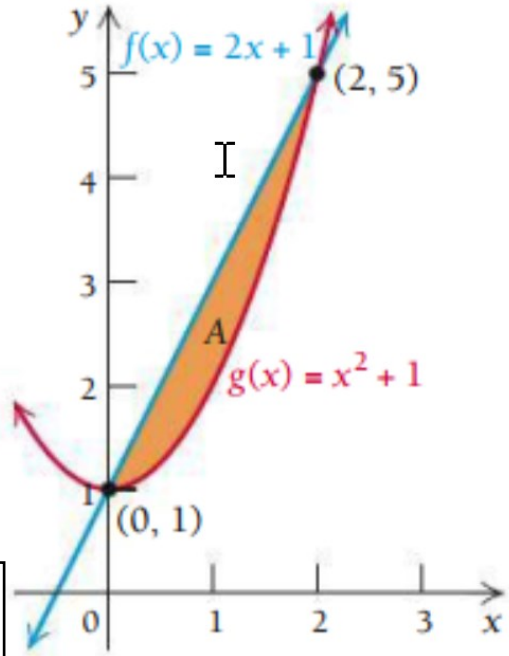


Area of a Region Bound by Multiple Graphs

$$\int_a^b \big(f(x) - g(x)\big)dx = \int_a^b f(x)dx - \int_a^b g(x)dx$$



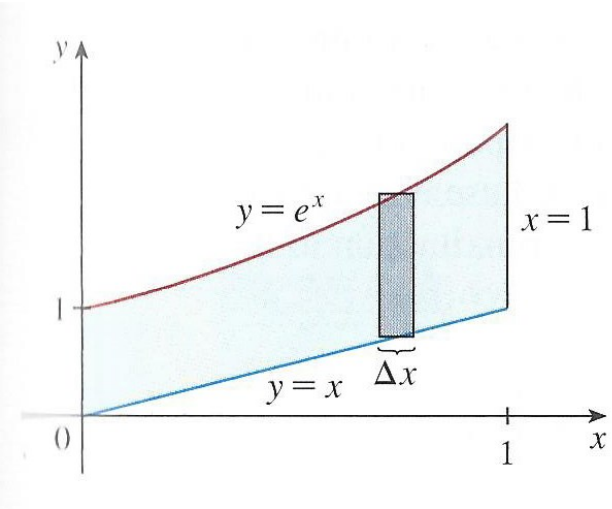
ex. Find the area of the region bound by the graphs $f(x) = 2x + 1$ and $g(x) = x^2 + 1$.



$= \boxed{\frac{4}{3}}$

Area Between Two Curves – Example #2

ex. Use an integral to find the area of the region bounded above by $y = e^x$, bounded below by $y = x$ and bounded on the sides by $x = 0$ and $x = 1$.



$$= \boxed{e - \frac{3}{2}}$$

$$\int_a^b \left(Y_{Top} - Y_{Bottom} \right) dx$$

Area Between Two Curves – Do

ex. Find the area of the region bounded by $y = x,\ y = x^3, x = 0,\ x = 1.$

$$\int_a^b (Y_T - Y_B)\,dx$$

$$= \boxed{\frac{1}{4}}$$

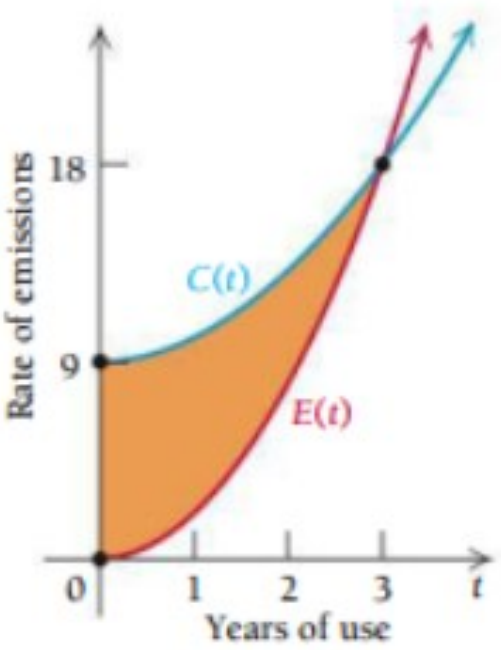
Area Between Two Curves – Application

ex. A student develops an engine believed to meet state emission standards.

The new engine's emission rate is given by $E(t) = 2t^2$

where E is in billions of pollution particles per year and t is time in years.

The emission rate of a conventional engine is given by $C(t) = 9 + t^2$.



At what time will the emission rates be the same?

$$\int_a^b \left(Y_T - Y_B\right) dx$$

What reduction in emissions results from the use of the student's engine over first 3 years?

$= \boxed{18}$

Interpretation: